Orientation-ocularity maps: A technique for computer vision

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Abstract

Based on the workings of visual cortical area V1, a model for an architecture for early computer vision is proposed. We propose to do image processing for computer vision on the basis of a combined map, of edge orientation and ocular-dominance, rather than on the basis of an edge map alone. In particular, the combined representation of edge orientation and ocular dominance is proposed for the computation of stereoscopic disparity.

Introduction

At cortical visual area V1, the attributes of edge orientation (that we code as $\phi \in [0, \pi)$) and ocular dominance (that we code as $\eta \in [-1, 1]$, -1 for left and 1 for right) are made *explicit* [24]. These attributes are superimposed on a retinotopic mapping, the retino-cortical mapping, a nonlinear yet angle-preserving map of the type $\log(1+z)$, approximately linear near the fovea and approximately logarithmic at the periphery.

Retinal ganglion cells fire in response to less sophisticated stimuli than V1 cells do, cortical area V1 making explicit attributes of the visual world that are not made explicit at the ganglionar level but that are based on them. V1 cells work on the basis of ganglionar output, and process further the information originated at the receptoral level that has been distilled by the retina. In between the retina and the cortex, there is the intermediate station of the thalamus; cortical responses are based on thalamic responses which in turn are based both on retinal and cortical inputs. Among the visual attributes made explicit in the retina are both luminance and chrominance edges, and among the attributes made explicit at V1 are edge orientation and ocular dominance, two attributes surely used to compute stereoscopic depth. The two main types of cell at V1 are *simple cells* and *complex cells*. Each simple cell responds with a certain degree of ocular dominance and to a certain edge orientation, within a nearly rectangular receptive field with two or three on/off regions separated by longitudinal straight lines. Complex cells also have the attributes of ocularity and orientation but do not have an on/off structure, responding to an appropriately oriented line that falls anywhere in the receptive field. Cells with a given degree of ocular dominance and cells that respond to a given orientation determine tangential line paths on V1 that meet orthogonally. Although this organisation is not universal, being different in squirrel monkeys and in some humans [1], this cortical relationship between edge orientation and binocularity suggests that it may be wise to consider edge orientation within a variety ocular dominances, when computing stereopsis.

Even though the lateral geniculate nucleus (LGN) of the thalamus, after the optic chiasm, provides the first anatomical opportunity for the appearance of binocular cells, where axons from both retinae synapse with proximity, and also in spite of the cortical input to the thalamus, thalamic cells remain monocular. It is likely then that the thalamic cortical input is only modulatory. There is no major feedback from the cortex or thalamus back to the retina (although something like that occurs in cephalopods [6]), but there is a big cortical input to the LGN [38]. Anatomically, within a layered structure of six retinotopic yet monocular maps, the LGN separates the parvocellular, magnocellular and koniocellular systems. Thalamic receptive fields are pretty much like those of the ganglion cells: monocular, with circular receptive fields, most with a center/surround spatial structure.

The visual cortex is also 6-layered but unlike the thalamus the vertical communication between cells is heavy, and at V1 there is only one, two-to-one, retinotopic map. Locally, the cortex shows edge orientation preferences[1] that, tangentially to the surface, mostly varies smoothly but that also includes point singularities (called pinwheels) as well as line discontinuities (called fractures). A surface pattern of orientation preferences, or *planform* [9] results that has been studied by means of dynamic system techniques [42]. Likewise, there is a planform of ocular dominance slabs, that is *orderly placed* with respect to the planform of the orientation columns. The two-to-one mapping of the visual field to the cortex is such that the cortex smoothly[2] alternates its responsiveness to the left-eye and to the right-eye; cells at the medians of the ocularity slabs being monocular and cells at the boundary of the slabs being fully binocular and cells at other places having different degrees of ocular dominance.

The 3D scene being watched is projected twice, once onto each retina and, under normal circumstances, the projections are *similar*. The *visual field* is a somewhat ambiguous concept referring to a combination of the 2D retinal projections. Next, you have two maps of the right visual field to the left cortex and two maps of the left visual field to the right cortex; the two maps give slightly different results and different V1 cells fire, depending on the contents of the 3D scene. Corresponding to a line in the 3D scene with a slant that changes its depth, the orientation of the retinal projections is different and the corresponding monocular firing cortical cells (e.g. if of the *simple* type) will be of different orientation types.

Consider the following notion of a virtual *backwards projection* from the ganglion cells onto the receptoral layer, in order to obtain retinal and cortical *thick lines*, for the modelling the workings of V1. Corresponding to a line in the 3D scene being watched there is a "thin" projection on each retina, and corresponding to each of these retinal projections consider a virtual "thick line" that engrosses the actual thin projection of the 3D line; the thick line results by considering all receptors in the receptive fields of

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[1] Exceptions to this orientation preference rule are cells at the 4th layer with receptive fields like those of thalamic cells and cells in the upper layers at the cytochrome oxidase blobs, that also have center/surround circular receptive fields, of the double opponent type.

[2] At the sublayer 4C of V1, where the thalamic axons arrive, the cells are all monocular and you have a neat interlacing of cells with exclusive provenance from the left-eye and from the right-eye.

the ganglion cells that are firing in response to the line[3]. As the two (left and right) retinal thick lines are mapped via the retinocortical map into the V1 cortical areas, two thick lines result in the cortex, that will cover cortical cells of different orientations and different ocularities. The two cortical thick lines will not coincide in general and, of the cortical cells covered by each thick line, only those of an appropriate ocularity and an appropriate orientation will fire. We propose that in order to be a unique fused perception of the 3D line, these cortical thick lines must have a nonempty intersection that includes binocular cells. The thickness of the lines decreases near the foveal region Alternatively, you could map back the ocular slab planforms, as is done in Figure 4 of [2], as well as the orientation planforms, into the retinal visual field, and paint the retinal thick lines on top of them.

Unlike the case of a cyclopean animal, or the typical single-lens camera computer vision system, the input to our visual nervous system consists of two retinal projections of the 3D scene being watched and, at V1, the two retina-processed projections are interlaced in a system of ocular dominance slabs; yet, it is a fact that we perceive a single, mostly fused, view of the 3D world around us, that somehow should correspond to a hypothetical early-processed 2D visual field. Unlike insects with their compound eyes, vertebrate and cephalopod eyes are "simple" in the sense of having a single aperture for light to go in and fall on a retina, as in a stenopeic camera. The addition of a lens (not present in the nautilus) with a varying refractive index complements the optics of the pupil and improves the focusing power for a given eye size and shape. The pupil radially inverts the 2D projections from the cornea to the retina, inverting in particular both the up-down and left-right dimensions. It is also the case of all vertebrates to have a contralateral organisation of the forebrain with the corresponding left-right decussations of nerve fibers between the forebrain (including the hemispheres but not the cerebellum) and the sensory-motor organs (excepting olfaction) and muscles in the rest of the body, at several places in the nervous system. It has been argued that this contralateral organisation evolved to account for the left-right inversion of the visual field at the retinae, making use of Ramon y Cajal's [20] three principles of neural economy (of space, of material and of time). Other reasons for the evolution of the contralateral organisation of the forebrain have been given as well [22].

The role of the optic chiasm, where there are no synapses, is to decussate half[4] of the ganglion axons of each retina to allow the right hemisphere process the left visual field (the right-half parts of retinas) and viceversa; the lateral periphery of the visual field (the nasal retinas) is mapped to the lateral parts of V1 (near V2), the point of fixation and convergence is mapped near the medial part of the brain where neurons synapse through the corpus callosum, and the upper part of V1 processes the lower part of the visual field (the upper retinas) and viceversa; in this way, that the map from visual field (and from each retina) to the cortex is *orientation* preserving. All vertebrates, and also the cephalopods, have nerve chiasms that swap, partially or completely, the output of the retinae, before feeding it to the brain (interestingly enough, Newton[5] thought about this possibility [26]). Birds (which interestingly enough do not have a corpus callosum) decussate the axons within the brain, at the visual Wulst, a point of view held by Descartes for man. Still, another possibility that seems not to have evolved is one that avoids the contralateral organisation of the forebrain and uses an optic chiasm but involves four swaps of bundles of fibers; in this case, the nasal axon retinas do not switch sides but the temporal axons do.

Lists of principles, or what might be called axioms, have been made [27], that the planform of orientation columns must obey [28]; however, no principle regarding a relationship with the planform of ocular dominance columns is usually included, and thus such collections are surely incomplete. In [27], it is stated "It is possible that interactions between sensory features (like ocularity and orientation) induce corresponding interactions between the feature maps which allow only one of these maps to be described by these simple principles." It is likely that there may be orientation planforms that meet a given set of criteria, such as almost everywhere continuity, *local surjectiveness*[6] and homogeneity, that are not compatible with normally observed patterns of ocular dominance. In fact, a reason for the specific geometric interaction between the two patterns has not been given.

Each simple cortical cell responds with a certain degree of binocularity, to a certain edge orientation and at a certain location on the visual field. The manifold of orientation-ocularity (ϕ, η) is the cylinder $[0, \pi)^\circ \times [-1, 1]$, a toppological annulus. If instead the manifold of ocularities is considered to be a also a circle, considering that there are two ways of going from one case of monocularity to the other, the orientation-ocularity manifold is a torus. In the case of an ocularity circle, you have two types of binocularity: the AND type and the OR type; with OR binocularity, the cell responds when the appropriate stimulus is presented to any eye; with AND binocularity, the cell respond only when the right type of stimulus is presented to both eyes; you have monocularity at two opposing poles and full binocularity at two opposing equatorial points.

The cortex gives a flat discrete approximation to the Cartesian product of position[7] (call it A) in the visual field and the manifold of ocularity and orientation (call it B). Subsets of this Cartesian product are *graphs* $G \subset A \times B$ of *relations* between positions and ocularity-orientations, in particular they may be considered to be graphs of ocularity-orientation functions $f : a \subset A \to B$ defined on subsets of the visual field position, i.e. the positions of edges. It is likely that by means of a cortical feedback modulation process you can select cells responding, say, to a given orientation, in

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[3] The receptive fields of ganglion cells are well modelled as differences of Gaussians, which resemble second derivatives of Gaussians; as Marr pointed out, the convolution with the second derivative of a Gaussian is equivalent to the second derivative of the convolution with the Gaussian. This convolution with a Gaussian, or blurring comes to mind when talking of this thickening of the line.

[4] In animals with no overlap of the left and right visual fields, the decussation is complete.

[5] Incidentally, Newton built his telescope using a mirror, instead of a lens as Galileo had done, avoiding chromatic aberration. Unlike a lens, a mirror inverts the geometric orientation of the 3D world we see trough it.

[6] By local surjectiveness we refer to the property that, for a small but big enough region, of about 1 mm^2, called *module* by Hubel and Wiesel (according to Hubel and Wiesel, the electrode must be tangentially advanced 2 mm across the cortical surface to locate cells without overlapping receptive fields), all orientations and all degrees of ocular dominance appear.

[7] For reasons mentioned later on, this coding of position in V1 is of *fuzzy position*. It remains unclear why we see things as neatly located in space.

what might be considered a function from orientation to position of sorts.

Receptive fields increase in size along the visual path; thus, the receptive field of a ganglion cell is large enough to contain an edge segment and the receptive field of aV1 cell is larger than that of a ganglion cell but small enough to contain only one orientation of an edge. The retinocortical map is not a 1-1 map in the sense that neighbour cortical cells usually have overlapping receptive fields. This overlap gives a continuity to the actual (discrete) receptoral projection and the points where such overlap is disrupted (neighbour cortical cells with disjoint (but abutting) receptive fields) indicate a loss of continuity. The continuity of our visual perception surely is based on the overlapping of receptive fields.

An *edge map* which is the name given to a joint representation of edge presence and location, is computed by the retinal, ganglionar layer; likewise, and a joint representation of position and orientation, or *orientation map*, is made explicit in cortical area V1. There is an inherent uncertainty in the position of an edge of a discrete image [33]; an edge between two regions of different luminance or chromaticity, rightfully belongs to both regions. Even less local is the case of the orientation of an edge: you need at least two edge points in order to get an orientation. Canny [10] edge detector works of the basis of the gradient of the image and thus detects simultaneously edge position and orientation. People work most of the time on the basis of an edge map (from which you can compute e.g. a Hough transform) and seldom on the basis of an orientation map; in addition, the quantisation of digital images may be an obstacle for the correct computation of orientation [15].

The retinocortical map

From 10^8 photoreceptors in the retina (from which only about 10^6 are cones), only 10^6 ganglionar axons leave the eye. ”Nowhere else in the visual system is the scene represented with as few neurons as in the optic nerve” [25][8]. At V1, the number of cells is again similar to the number of photoreceptors. The distribution of cone photoreceptors on the retina has a much higher concentration in the fovea (where there is a rod scotoma) than in the periphery. Likewise, ganglion cells with receptive fields centred near the fovea have a smaller size than those at the periphery; this receptor concentration gradient could be a substratum for the phenomenon of attention. The number of receptors corresponding to these receptive fields increases, from only one in the fovea (in the case of the midget ganglion cells) [18], to thousands in the periphery; so that the gradient of receptor-field sizes is much larger than the gradient of receptor concentration.

By interpreting both the retinae and the tangential surfaces of the V1 cortical regions as portions of the complex plane, a log map results [12]. For illustration purposes, using complex numbers, we simplify Schwartz's [37] formula $\log(z+a)$ for the retinocortical map, as being of the form $\log(1+z)$. This map provides a smooth transition from the linear identity map $f(z)=z$, for very small $|z|$, to the log map $f(z)=\log(z)$, for large $|z|$. The map $\log(1+z)$ is not only continuous but also conformal, that is, it preserves angles; see Figure 1. In [9] under the so-called *double retinocortical map*, (straight segments of) a logarithmic spiral at the retinal level is (are) mapped into (straight segments of) a set of parallel lines at the cortical level.

Writing $z=x+jy=|z|e^{j\angle z}$, the right visual field corresponds to $\angle z \in [\frac{-\pi}{2},\frac{\pi}{2}]$ and the left visual field to $\angle z \in [\frac{\pi}{2},\frac{-\pi}{2}]$; they are processed respectively by the left and right V1 cortical areas. For large $|z|$, lines that are radial in the visual field become parallel[9] in the cortical area and concentric circles in the visual field become also parallel lines, that are orthogonal to the images of the radial lines, see Fig. 2. So, in a geodesic sense, radial lines and concentric circles centred at the fovea are also made explicit in V1. The equation of the line $x=1$ in polar coordinates [40] is $r=\frac{1}{\cos\theta}$ and the equation of the line $x=L$ is $r=\frac{\log L}{\cos\theta}$, and rotations of this line give $r=\frac{L}{\cos(\theta-\theta_L)}$. The equation of the circle of radius one and centre at $x=1/2$ is $r=\cos\theta$. At the periphery, where the retinocortical map is approximately logarithmic, a rotation translates into a vertical displacement, and an expansion/contraction translates into a horizontal displacement. This provides for a constancy of representation under scaling and rotation that may be useful in recognition tasks.

At the fovea, where the map is nearly linear, the size of a receptive field of a V1 cell is about one quarter of a degree by one quarter of a degree, about the same size of the smallest ganglion receptive fields, at the far periphery, the sizes are about one degree by one degree. In addition to being a conformal map, the logarithmic map allows for small cortical regions of a given size to represent areas in the retina proportional to the sizes of the receptive fields of cells in the region, which are small near the fovea and larger at the peripheral retina. Using the inverse map e^w-1, the d ocular-dominance layout can be inverted, as it has been done in [2], to see how the ocular dominance planform looks from the point of view of the visual field.

The retinas are nonuniform due to the nonuniform density of the cones and rods. The ganglion layer is not uniform in the sense that the ganglion receptive fields vary in size depending on eccentricity; thanks to the retinocortical map, V1 is homogeneous with respect to the visual field, as it is seen in Roger Tootell's [17] radioactive 2-deoxyglucose image.

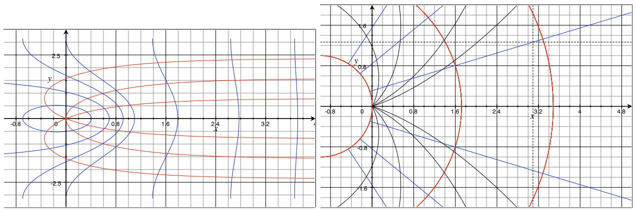

Figure 1. *Left, in blue, the image of circles of radii 0.5, 1, 1.5, 2, 5, 15, 40; in red, images of of rays from the origin at angles* $\pm\frac{5\pi}{4}$, $\pm\frac{\pi}{2}$ *and* $\pm\frac{\pi}{4}$, *under the map* $\log(1+z)$*; as shown in Figure 2, the left-half plane of this figure, and portions of the right-half plane, are not used in the cortical map. At right, the (reverse) images of horizontal lines, in blue (for* $\Im w=\pm1,\pm0.7,\pm0.3$*), vertical lines in red (for* $\Re w=0,1,1.5$*), and rays (in directions* $1\pm2i,1\pm i,2\pm i,3\pm i$*) in black, of the cortical plane, under the map* $\exp(w)-1$.

In [9] it is considered not only the retinocortical map but

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[8] Still, there could be a time code of which we are unaware.

[9] Actually, as pointed out in [3], at the periphery, the images of rays converge and the images of circles become curved and shortened; nevertheless, as a rough guide, $\log(a+z)$ is a simple and enlightening formula, mainly for nonperipheral regions.

also the way the orientation of edges in the retina are mapped to the orientation of lines in V1, in what it is called the *double retinocortical map*.

On Orientation and Ocularity

As you move tangentially in cortical area V1, cells respond to different edge orientations with different tuning bandwidths, and with different degrees of binocularity. Using image processing techniques, the subtle changes of reflectance of oxygenated blood corroborated at a global level the local measures of Hubel and Wiesel, and the ensuing use of colour to visualise orientation in [7] gave an early example of the visualisation of a cyclic magnitude (orientation) using hue in a homomorphic way [34].

Topologically, there is only one closed 1-manifold and the space of orientations is a topological circle; it is an annulus if the property of tuning is taken into account as radial distance in a polar plot [11]. Disregarding tuning, he combined manifold of orientation and ocularity is a 2-torus. The ocular dominance and the orientation planforms have comparable scales; ocular-dominance columns run in slabs 400 μm wide and most cells at or near (within 50 μm) a pinwheel of the orientation planform are sharply tuned.

Away from pinwheels, the border of an iso-orientation region lies orthogonal to the direction of highest variation of orientation; likewise, the border of an ocular-dominance column lies perpendicular to the direction of highest variation of ocularity. Also, directions of large variation of the ocularity tend to keep constant the orientation and viceversa. Pinwheels tend to be either monocular or of balanced binocularity (more than 80% are monocular in the case of the macaque), they tend to be placed near on the median of ocularity slabs. Crossing the line of balanced binocularity, the transition from a pinwheel to an adjacent one occurs along a path of smooth change of ocularity. Adjacent pinwheels of opposite ocularity are concordant in the sense that they frequently have opposite *chirality* (clockwise, counterclockwise). Yet, a 2×2 arrangement of clockwise pinwheels may have a neighbour counterclockwise pinwheel at a line of balanced binocularity. The majority of the non-monocular pinwheels are located close to ocular-dominance borders, where cells are binocular [11], [4]. At pinwheels, contrary (i.e. orthogonal) orientations correspond to disjoint but abutting retinal regions [14]. There are many cells on the median of ocularity slabs, that are therefore monocular, but are not pinwheel cells [13]. In strabismic cats the lines of highest binocularity correspond to zones of continuous variation of orientation preference, as opposed to pinwheel singularities [21].

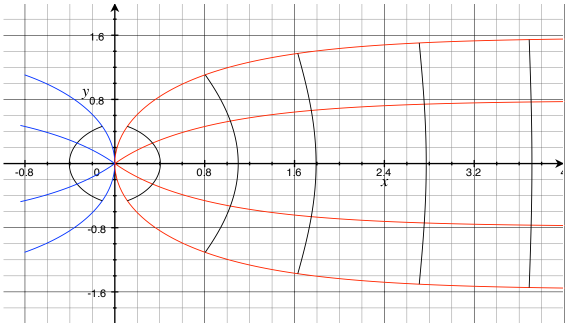

Figure 2. *The cortical map at the right hemisphere results from rays leaving the origin (central fovea) of the (left) visual field, with angles ranging between* $\frac{\pi}{2}$ *and* $\frac{-\pi}{2}$*, shown as the bottom and top red lines; the intermediate red lines correspond to directions at* $-\frac{\pi}{4}$ *and* $\frac{\pi}{4}$*. The black lines correspond to circles in the visual field, of radii 0.5, 2, 5, 15 and 40 (for the simplified* $\log(1+z)$ *map). For the left hemisphere there correspond the rays with direction angles between* $-\frac{\pi}{2}$ *and* $\frac{\pi}{2}$*, shown as the bottom and top blue lines. Each pair of points, at a given height, of the top and bottom* $\pm\frac{\pi}{2}$ *lines, at left and right (red and blue) is connected via the corpus callosum, so there is only one cortical connected V1 surface.*

Stereopsis

The vision systems vertebrates with overlapping left- and right view fields allow for the perception of depth at central directions of sight via stereopsis; this is mainly done on the basis of the disparities between the images on each retina. Yet, a vivid perception of depth under movement conditions occurs in Pulfrich stereo, where the visual stimulus consists of a single flat image and there is only a difference of intensity between the two images; here is no consensus [5], [31], [32] as to whether there are cells responding simultaneously to motion and disparity, to explain this phenomenon. It is possible to perceive depth from stereopsis without conscious fusion in what is called coarse stereo [29]. The region in the 3D scene that leads to fusion is called Panum's area.

You may speak both of position disparities and orientation disparities, both are related to depth, the concepts of disparity and of correspondence being interdependent. Under normal conditions [30], the images at the receptoral layers of the retinas are only slightly different and by inferring correspondences between different tokens made explicit further down the visual path, you speak of *disparities* between the images. It is likely that cells responding to disparities are binocular; yet, at V1, on a horizontal line, running from the fovea to the nasal periphery, the zone corresponding to the blind spot of the eye that is contralateral to the corresponding hemisphere is processed only by the ipsilateral hemisphere. Also, the parietal periphery of the retinal field is processed only by cells with receptive fields in the contralateral eye.

The geometries of visual space and of cortical space are related. Consider a horizontal plane in space that passes through the pupils and contains the fixation point; on this plane, consider the two lines that pass through the pupils P_L ad P_R and through the fixation point Q. Extend the lines well after the fixation point and, on the other side, until they intersect the retinas. These two lines divide the visual space in the plane, into four regions to be labeled *central near*, *central far*, *left* and *right*. The two central regions determine a double cone. The vertex of the double cone project on the foveas and points in the double cone project either nasally (if in the *central far* region) or parietally (if in the *central near* region) in the retinas, with the consequence that they are mapped to different talami and different hemispheres. Points off the double cone, in the *righ* and *left regions*, project nasaly *and* parietally into the contralateral middles of the retinas and, after the optic chiasm, in the contralateral talami and hemispheres. So, the matching of the projections of points off the double cone occurs within the same hemisphere and the matching of the projections of points in the double cone requires the use of the corpus callosum. In any case, a point different from the fixation point will produce two projections that should be made to correspond. Also, a pair of points determines a line in space, if this line does not fall entirely on the boundary of the double cone, the projection of a pair of points will be a set of four points that potentially give rise to two possible matches of which one is to be chosen. For *n* points, with no occlusion, the number of possible matches is $n!$ to be reduced to *n* actual matches. Each pair of matched points determines an orientation in visual space.

Positional Disparity from Depth Gradient

We derive a formula for the relation between position disparity in the retinae, and difference of depth. We work on a single horizontal xy-plane, disregarding the vertical z axis; assume you have a parallel stenopeic camera pair, with pupils at xy-cordinates $(-A,0)$ and $(A,0)$. For this set up, the horopter is straight. For simplicity, assume that the focal plane is in front of the pupils, at distance f along the y direction, instead of at the back. Given the points (X,Y) and (X',Y'), with $Y,Y' > 0$, you want to relate the difference $Y-Y'$ of depths with the disparity of their projections on the focal planes or retinas. The points project at points $(-X_L,f)$ and $(-X'_L,f)$ for the (left) camera at $x=-A$, and (X_R,f) and at points (X'_R,f), for the (right) camera at $x=A$. Let $\Delta_L = X'_L - X_L$, $\Delta_R = X'_R - X_R$ and $\partial = \Delta_L - \Delta_R$ be the position disparity. Here we assume the correspondence problem to have been solved. By similarity of triangles you have

$\frac{Y}{X+A} = \frac{f}{X_L+A}$, and, $\frac{Y'}{X'+A} = \frac{f}{X'_L+A}$ and

$\frac{Y}{A-X} = \frac{f}{A-X_R}$, and, $\frac{Y'}{A-X'} = \frac{f}{A-X'_R}$ so that

$\Delta_L = \frac{f}{Y'}(X'+A) - \frac{f}{Y}(X+A)$ and

$\Delta_R = \frac{f}{Y}(A-X) - \frac{f}{Y'}(A-X')$ therefore

$\partial = 2fA(\frac{1}{Y'} - \frac{1}{Y})$

The equation does not give a linear relation between the disparity ∂ and difference of depth $Y-Y'$ yet, if you are interested in qualitative information such as knowing which of the two points is nearer, it is of course useful.

Orientation Disparity from Depth Gradient

We derive a formula that tells how the orientation changes from the view point of one eye to the other as a function of depth. Again, assume that a parallel pair of stenopeic cameras is used; also, assume that $f=1$ i.e. that the pupils are positioned at $(\pm 1,0,0) \in \mathbf{R}^3$ and that the projection, focal plane is at $y=1$. Rays from the points $(-1,0,0)$ and $(+1,0,0)$ to a generic point (x,y,z) intersect the focal plane at

$(\frac{x+1}{y} - 1, 1, \frac{z}{y})$ and $(\frac{x-1}{y} + 1, 1, \frac{z}{y})$, respectively.

Let $(x_0,y_0,z_0) + \lambda(x_1,y_1,z_1)$, $\lambda \in \mathbf{R}$ be a line in three space. The projections of the line on the xz focal plane (at $y=1$) are then (dropping the y coordinate)

$(\frac{x_0+\lambda x_1+1}{y_0+\lambda y_1} - 1, \frac{z_0+\lambda z_1}{y_0+\lambda y_1})$ and $(\frac{x_0+\lambda x_1-1}{y_0+\lambda y_1} + 1, \frac{z_0+\lambda z_1}{y_0+\lambda y_1})$

To obtain the slopes of the two projections, consider the values 0 and 1 of λ; the following two pairs of points of the xz plane result:

$(\frac{x_0+1}{y_0} - 1, \frac{z_0}{y_0})$, $(\frac{x_0+x_1+1}{y_0+y_1} - 1, \frac{z_0+z_1}{y_0+y_1})$ and

$(\frac{x_0-1}{y_0} + 1, \frac{z_0}{y_0})$, $(\frac{x_0+x_1-1}{y_0+y_1} + 1, \frac{z_0+z_1}{y_0+y_1})$

from each pair of points you get the slope of a line. The slopes are

$$\frac{\frac{z_0+z_1}{y_0+y_1} - \frac{z_0}{y_0}}{\frac{x_0+x_1+1}{y_0+y_1} - 1 - (\frac{x_0+1}{y_0} - 1)} = \frac{y_0 z_1 - y_1 z_0}{y_0 x_1 - y_1 x_0 - y_1} \text{ and}$$

$$\frac{\frac{z_0+z_1}{y_0+y_1} - \frac{z_0}{y_0}}{\frac{x_0+x_1-1}{y_0+y_1} + 1 - (\frac{x_0-1}{y_0} + 1)} = \frac{y_0 z_1 - y_1 z_0}{y_0 x_1 - y_1 x_0 + y_1} .$$

Thus, even though in general the slopes of the two projections of the line are different, whenever the y-direction y_1 of the line is zero (and the line lies on a plane $y = \mathit{constant}$), the two slopes are the same; in fact, the quotient of the slopes is given by

$$\partial_s = \frac{y_0 x_1 - y_1 x_0 + y_1}{y_0 x_1 - y_1 x_0 - y_1} = \frac{y_0 x_1 - y_1 (x_0 - 1)}{y_0 x_1 - y_1 (x_0 + 1)}$$

and you have that being on a plane $y = \mathit{constant}$ is a necessary and sufficient condition for the line, for the slopes of the two projections to be equal. The y-direction is the direction *ahead* from the computer vision system, or binocular animal. So, only for this particular type of slant, when the depth of the line changes, the left-eye and right-eye retina *orientations* are different. Therefore, a difference in orientation corresponding to edge segments known to correspond (and that thus should be perceptually fused) signals depth, and there must be a certain correspondence between the depth disparity ∂ of two edge segments being matched, and their orientation disparity ∂_s. Thus, there is a relationship between position disparity in the receptive fields in the retina and the ratio of the orientations that should be exploited in stereo vision algorithms.

More on stereopsis

For random dot stereograms [39], two points falling within the excitatory band of a simple cell will de facto signal, if weakly, a line segment, making the cell respond. In a random dot stereogram the points are approximately uniformly distributed and there will be only such weak responses, that will not be obscured by stronger responses.

If a 3D line segment being watched has a slant in the y direction its depth changes and the corresponding left-eye and right-eye *virtual thick cortical lines* (see the Introduction) will fall on different positions and will be of different orientations. Binocular cells that fall on the intersection of the two thick cortical lines, with orientation tuning (bandwidths) large enough, will respond to both projections. If the thick lines do not coincide on top of a binocular cell, we conjecture that then the line segment will be perceived as double.

Some V1 cells make position disparity explicit [19]. "The amount of horizontal illpositioning, or disparity, that can be tolerated before the response disappears is a fraction of the width of the receptive field" [17]. A lack of concordance between the position disparity and the orientation disparity could be used to discard false position correspondences. It may be the case that two near by monocular cells with overlapping receptive fields signal each a different orientation and then, that these two cells feed into a third cell at a further processing stage, with the difference in orientation being made explicit; also, it is likely that a short segment of binocular cells in between the two monocular cells will fire and feed the further stage. Since the stereo matching of cells with overlapping receptive fields occurs for cells of opposite ocularity, they are cells in different yet neighbour ocularity slabs.

In a continuum of orientations, nearly monocular cells are located at the median of the ocularity slabs and nearly binocular cells are located at the boundaries of the slabs. Cells of opposite ocularity cannot be neighbours, lying in neighbour slabs or perhaps farther; they may have overlapping receptive fields. Cells of similar orientation and with overlapping receptive fields lie in about the same orientation column; being near by, they will have a similar colour in the orientation planform.

Conjecturing that at the center of gaze there is no disparity [35], [36], at the area of V1 corresponding to the central foveal region, near the border of V1 that is connected via the corpus callosum with the contralateral V1 area, left- and right- visual-field edges at the corresponding cells of V1 will signal the same orientation, even if they are finely tuned.

Conclusion

The way the phenomenon of stereopsis is usually approached and the way we engineer machines that exploit the phenomenon is top down, knowing that there are unique tokens in the 3D visual scenes that produce so-corresponding retinal projections that must be matched. The biological vision system on the other hand is a machinery that has evolved certainly in a bottom up fashion.

At cortical area V1, the first stage in the visual system where it is possible to talk about stereo disparities, the attributes of ocular dominance and of edge orientation are made explicit. Disparity must then be a relation between oriented edge segments, made explicit either by binocular cells, by further processing of monocular cells, or both.

An arrow in 3 space, the more pointed towards you, the larger the difference of orientations of the left-eye and the right-eye image. Corresponding left and right edge segments feeding a binocular cell that signals depth must have compatible positional and orientation disparities; that is, the difference of orientation and the difference of position in the retinal projections are interdependent, since each depends on the change of depth of the 3D segment.

Orientation is finely tuned at the center of orientation blobs [23]. The potential expression of the attributes of orientation and ocularity is diverse and you find a continuum of possibilities in between monocularity and balanced binocularity (yet, in layer 4C, cells are monocular), as well as orientation bandwidths [17] of several degrees, and of receptive field size. Nevertheless, it is conceivable that cells do respond only to a small set of orientations; think for example that as few as 3 types of a cone photoreceptor (L, M and S), broadly tuned, give rise to a rich perceptual manifold of colour.

A given cell is responsive to a variety of stimulus types (e.g. wavelength and orientation) and the fact that, for a given stimulus type, the cell responds within a given range of the corresponding physical parameter, rather than to a single value of it, and that for other neighbour cells these ranges overlap, is an ubiquitous property of the sensorial nervous system, that may well provide continuity with discrete machinery.

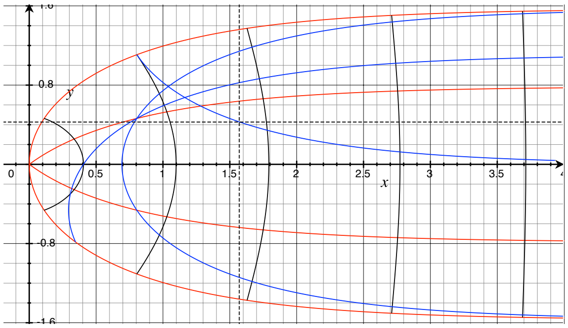

Figure 3. *The cortical map at the right hemisphere; black and red lines are as in Fig. 2. The blue lines correspond to the lines* $y = 2$, $x = 1$ *and* $y = -1 + 2x$, *in the right-half plane* $x \geq 0$, *of either, say, the left-eye retina or the right-eye retina.*

Wittgenstein [41] says he doesn't expect to exist a copy in the nervous system of the singular representation, the singular thought, or of memory. Consciousness should not be entirely the same as anything in the brain but something new. So, with colour, area 8 or 4 of Zeki may be closer to perception but not quite the same.

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